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On How Stochastic Magnetic Perturbations Influence Dynamics and Relaxation

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Why Now?

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- Syntheses of good confinement and optimal power handing drive us to 3D, often involve stochastic magnetic fields.

i.e.

- RMP: L→H transition threshold
  - turbulence persists, though modifed
  - flows, $\langle E_r \rangle$ and interactions modified → see C.C. Chen, this session and Invited, Tuesday PM.
- Stellerator
- Island Configurations – ITB
- Disruption Evolution

➔ What of turbulence structure?

Effects on Turbulence Structure?

- How does stochastic field modify structure and rates of turbulence?
- E.g. Ancient Classics: Kaw, Valeo, Rutherford '79 et. seq.
  - Tearing, in braided magnetic field
  - 'anomalous dissipation' by $\left\{ \begin{array}{l} \text{electron viscosity} \\ \text{hyper-resistivity} \end{array} \right.$ ⇨ rescue resistive MHD

    $E_{\parallel} = \mu J_{\parallel} - \mu \nabla_{\perp}^{2} J_{\parallel}$
  - Rather minimal micro ←→ macro connection
- Begs for a 'simple problem' for in-depth analysis

Model

- Resistive Interchange + Static, specified $\langle \tilde{b}^2 \rangle_{k'}$

$$(\partial_t - \nu \nabla_\perp^2)\nabla_\perp^2 \phi = -S\left(\nabla_\parallel^{(0)} + \vec{b}\cdot\nabla_\perp\right)\left(\nabla_\parallel^{(0)} + \vec{b}\cdot\nabla_\perp\right)\phi - \frac{g}{L_p}\partial_y P$$

$$\partial_t P - \chi \nabla_\perp^2 P = -\partial_y \hat{\phi}$$ χ, ν: TBD

- Key: $\boxed{\nabla_\parallel = \nabla_\parallel^{(0)} + \vec{b}\cdot\nabla_\perp}$ → parallel gradient along randomly tilted lines
- Stochastic PDE, with Multiplicative Noise:

  ala' Schrodinger Eqn. with random potential (c.f. Kraichnan, "random coupling")

$$-\nabla^2\psi + U_0(x)\psi + \tilde{U}(x)\psi = E\psi$$

- Generic: $\nabla\cdot\vec{J} = 0$ → $\nabla_\parallel J_\parallel + \nabla_\perp J_\perp = 0$

  wandering Lines...

- N.B. Presume equivalence of random perturbations and stochastic lines

Two Scale Formulation

- <u>Multi-scale</u> $|k'| \gg |k|$ $\bar{\phi}_k$ → test field

$$\phi = \bar{\phi} + \tilde{\phi}_{k'}$$

$b_{k'}$

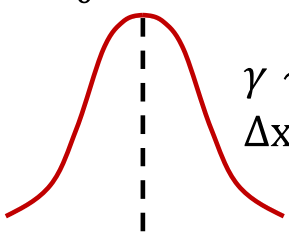
Key? → how determine

→ potential fluctuations generated on small scale to maintain $\nabla \cdot J = 0$

→ $\nabla_\parallel J_\parallel \neq 0 \;\Rightarrow\; \nabla_\perp \cdot J_\perp \neq 0$
Cells must accompany current convergences

- <u>Picture</u>

$\vec{k} \cdot \vec{B}_0 = 0$



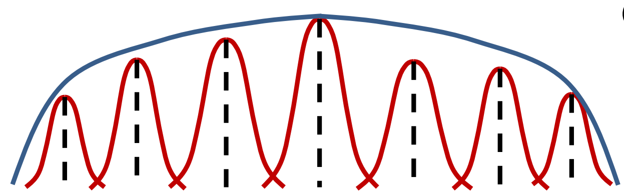
$\gamma \sim S^{-1/3}\tau_A^{-1}$
$\Delta \mathrm{x} \sim S^{-1/3} a$

low n

single test mode

+

Spectrum of prescribed static magnetic fluctuations



densely packed

$$|b_{k'}|^2 = |b_0|^2 S(k_\theta)\Gamma\Big((r - r_{k'})/w_{k'}\Big)$$

Insight from a Classic

S

- Two Classics: Rechester, Rosenbluth '78 - test particle picture

  Kadomtsev, Pogutse '78 – hydrodynamic $l_{mfp} < l_c$

- Elements:

$$\nabla \cdot \vec{q} = 0 \qquad \vec{b} = \vec{b}_0 + \tilde{b}$$

$$\vec{q} = -\chi_{\parallel} \nabla_{\parallel} T \, \hat{b} - \chi_{\perp} \nabla_{\perp} T \qquad \nabla_{\parallel} = \nabla_{\parallel}^{(0)} + \tilde{b} \cdot \nabla_{\perp}$$

$$\chi_{\parallel} \gg \chi_{\perp}$$

- N.B.: $\nabla \cdot \vec{q} = 0$ introduces element of self-consistency; prevents heat accumulation

- $\langle q_r \rangle = -\chi_{\parallel} \left[\langle \tilde{b}_r^2 \rangle \frac{\partial \langle T \rangle}{\partial r} + \langle \tilde{b}_r \nabla_{\parallel} \tilde{T} \rangle \right] - \chi_{\perp} \nabla_{\perp} \langle T \rangle$
- $\tilde{T}$ from $\nabla \cdot \vec{q} = 0$ ➔ cancellations, significant deviation from test particle theory due $\nabla \cdot \vec{q} = 0$

Two Scale Formulation, cont'd

$\phi = \bar{\phi} + \tilde{\phi}$

$\bar{\phi} \rightarrow \vec{k}$ envelope

$\tilde{\phi}, \tilde{b}_{k'} \rightarrow \vec{k}'$

Envelope:

$$(\partial_t - \nu \nabla_\perp^2)\nabla_\perp^2 \bar{\phi} + \frac{S}{\tau_A}\, \partial_x \left|\tilde{b}_r\right|^2 \partial_x \bar{\phi} = -\frac{S}{\tau_A}\, \nabla_\parallel^{(0)2} \bar{\phi} - \frac{g}{L_P} \partial_y \bar{P} + \langle \nabla_\parallel^{(0)} \nabla_\perp \cdot \left(\tilde{b}\tilde{\phi}\right)\rangle + \nabla_\perp \cdot \langle \tilde{b} \nabla_\parallel^{(0)} \tilde{\phi} \rangle$$

① ② ③

Small Scale Fluctuation: Inversion → $\boxed{G(x, x'')}$

$$(\partial_t - \nu \nabla_\perp^2)\nabla_\perp^2 \tilde{\phi}_{k'} + \frac{S}{\tau_A}\, \nabla_\parallel^{(0)} \tilde{\phi}_{k'} + \frac{g}{L_P}\, \partial_y \tilde{P}_{k'} = -\frac{S}{\tau_A}\left[\nabla_\perp \cdot \left(\tilde{b}_{k'} \nabla_\parallel^{(0)} \bar{\phi}\right) + \nabla_\parallel^{(0)}\left(\tilde{b}_{k'} \nabla_\perp \bar{\phi}\right)\right]$$

$\tilde{P}$ equation

What's the Physics? - what does this mess <u>mean</u>?

①

$$\frac{S}{\tau_A}\,\partial_x\,\left|\tilde{b}_r\right|^2\partial_x\bar{\phi}$$ → $\boxed{\text{magnetic vorticity damping}}$

→ 3rd order $\nabla_{\|}J_{\|}$

from

$$\sim\ \hat{b}\cdot\nabla_{\perp}\left(-\frac{1}{\eta}\hat{b}\cdot\nabla_{\perp}\right)\bar{\phi}$$

Re-express: $$\frac{S}{\tau_A}\left|\frac{\tilde{B}_{rk'}}{B_0}\right|^2 = \frac{V_A^2}{\eta}\frac{k_\theta'^2}{L_s^2}\,{w_I'}^4$$

$w_I' \equiv$ island width

for stochastic field

So: $$\frac{S}{\tau_A}\,\partial_x\,\left|\tilde{b}\right|^2\partial_x\bar{\phi} \sim \frac{V_A^2}{\eta}\frac{k_\theta'^2}{L_s^2}\frac{{w_I'}^4}{(\Delta x)^2}\bar{\phi}$$

<u>estimate</u>

→ $\bar{\phi}$ layer width

Magnetic Torque, cont'd

$(\nabla_\| J_\|)^{(1)} \sim \frac{V_A^2}{\eta} \frac{k_\theta^2}{L_s^2} (\Delta x)^2 \, \bar{\phi}$ → bending term, linear

Key question:

$(\nabla_\| J_\|)^{(3)} \sim \left(\nabla_\| J_\|^{(1)}\right)$ ←→ When will 3rd order magnetic torque balance first order

→ $\boxed{w_I' \sim \left[\frac{k_\theta^2}{k_\theta'^2} (\Delta x)^4\right]^{1/4}}$ → Width of small scale island needed

→ Reminiscent of Rutherford '73; but with $k_\theta^2 / k_\theta'^2 < 1$ factor, due multi-scale interaction

$(\nabla_\| J_\|)^{(2)} > \left(\nabla_\| J_\|^{(1)}\right)$ → magnetic torque supplants inertia in vorticity balance unambiguously stabilizing basic vortex flow of mode

E-fields along Perturbed Lines ②, ③

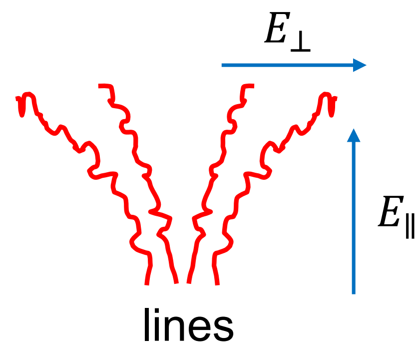
The Rest...

Consider:

②

$$\left(\nabla_{\parallel}^{(0)} + \tilde{b}\cdot\nabla_{\perp}\right)\left(-\frac{1}{\eta}\left(\nabla_{\parallel}^{(0)} + \tilde{b}\cdot\nabla_{\perp}\right)\right)(\bar{\phi} + \tilde{\phi})$$

③



$$③ = \langle \nabla_{\parallel}^{(0)}(\nabla_{\perp}\cdot\tilde{b}\tilde{\phi})\rangle = \langle -\nabla_{\parallel}^{(0)}\left(\underline{\tilde{b}_{\perp}\cdot\vec{\tilde{E}}_{\perp}}\right)\rangle$$

$$② = -\nabla_{\perp}\cdot\langle\tilde{b}\nabla_{\parallel}\tilde{\phi}\rangle = -\nabla_{\perp}\cdot\langle\underline{\tilde{b}_{\perp}\tilde{E}_{\parallel}}\rangle$$

E field projections along wandering lines

$\vec{\tilde{E}} = -\nabla_{\perp}\tilde{\phi}$, $\quad \tilde{\phi} \sim \tilde{b}\phi$

$\nabla_{\parallel}J_{\parallel} \neq 0$ at test wave resonance surface

→ Nonlinear Bending + Resistivity → Dissipative Nonlinearity

Screening, Small Scale $\tilde{\phi}$ and Convective Cells

How obtain $\tilde{\phi}$?

mean potential

$$L_{k+k'}\tilde{\phi}_{k+k'} = C\ \tilde{b}_k\ \bar{\phi}_k$$

→ ~ Langevin Eqn.

$$\left(\partial_t V + \frac{\gamma}{m}\ V = \frac{\tilde{f}}{m}\right)$$

eigen mode operator with $\underline{\nu, \chi}$

C.C.

noise/modulation (multiplicative)

Convenient to take
k → slow interchange
k' → fast interchange

$|k| \ll |k'|$:

$$L_{k'}\tilde{\phi}_{k'} = C\ b_{k'}\bar{\phi}$$

$$\boxed{\tilde{\phi} = \int dr''\ G(r, r'')C b_{k'}\bar{\phi}}$$

→ obtains $\tilde{\phi}$ via Green's function

Screening, cont'd - How determine $\tilde{\phi}$?

- Langevin Eqn. ←→ Fluctuation-Dissipation Theorem (?!)

$$\left|\tilde{\phi}_{k'}\right|^2 \approx \frac{|c|^2\,|b_{k'}|^2\left|\bar{\phi}\right|^2}{L_{-k'}L_{k'}}$$

$L_k \equiv$ operator

→ $k_\theta'^2 \gg k_\theta^2$

∴ fast interchange

~ stationarity → damped response

→ L must be $\boxed{\text{over-stable}}$

- ν, χ → turbulent diffusion from small scale electrostatic cells - $\vec{\tilde{v}} \cdot \nabla\nabla^2\phi$

$$\nu, \chi \;\rightarrow \nu_T \qquad \nu_T \sim \left(g/L_p\right)^{1/2} {k_\theta'}^{-2} + \delta\nu_{\mathrm{T}}$$

saturated

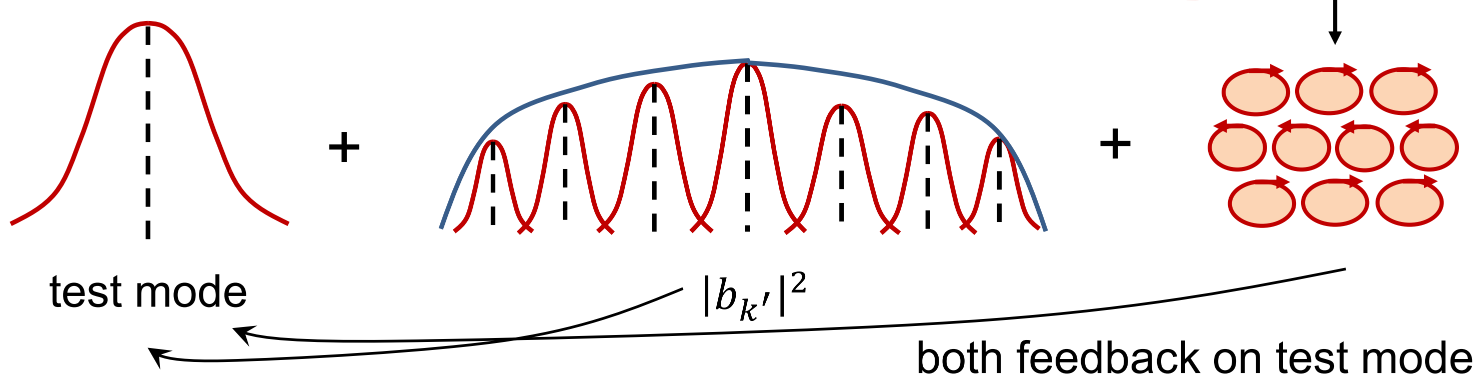
$\underline{\vec{V}} \cdot \underline{\nabla} P$

small increment added

$$\nu_T \approx \sum_{k'} |c_{k'}|^2 \langle \tilde{b}^2\rangle_{k'} \left|\bar{\phi}\right|^2 \gamma_{k'}^{-1} \Big/ \left[k_\theta'^2 - \frac{g {k_\theta'}^2}{L_p\left(\nu_T {k_\theta'}^2\right)^2}\right]^2 \Rightarrow \underline{\delta\nu_T}$$

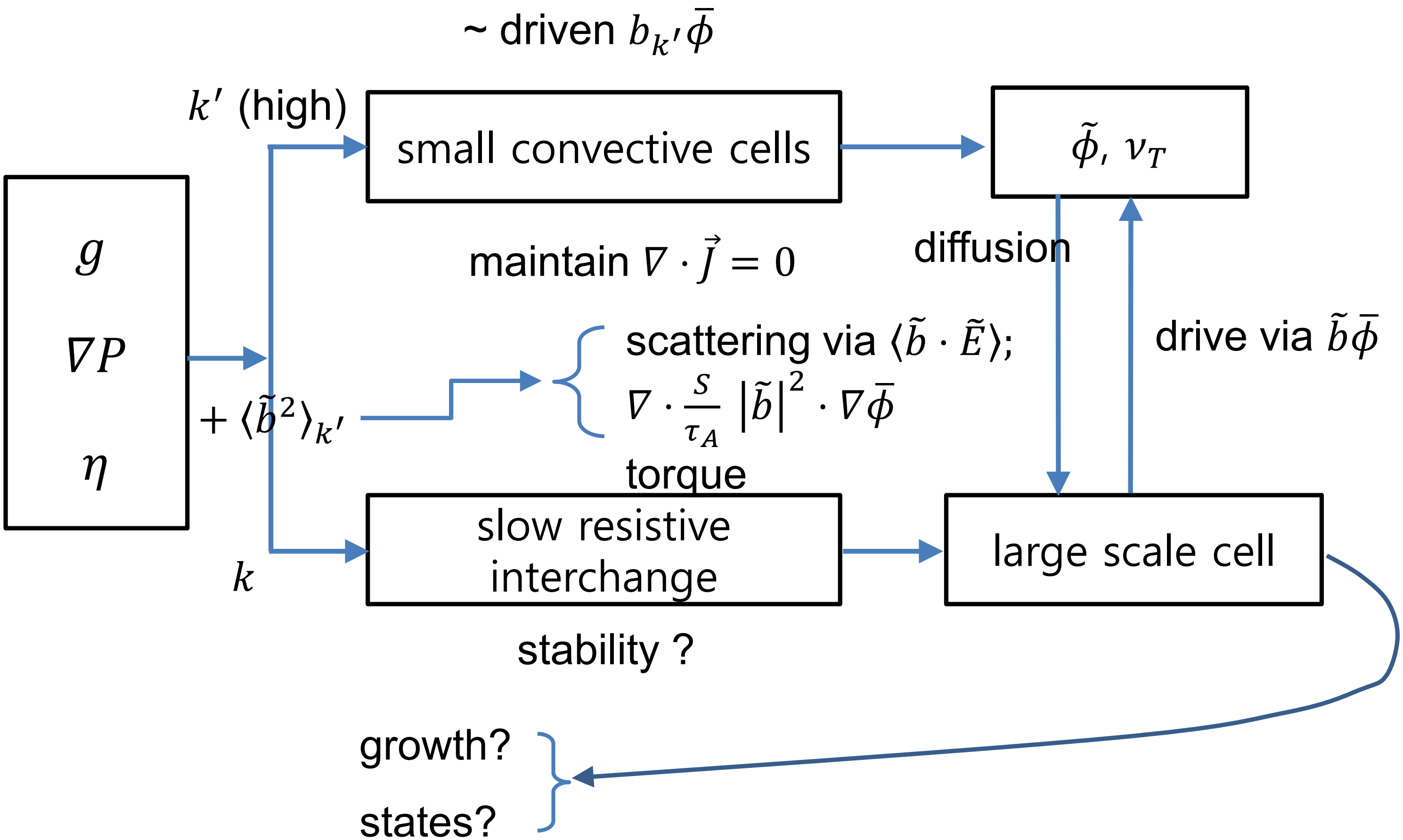
Screening, cont'd

- System <u>really</u> is : small scale cells driven



- $\nu, \chi \;\; \rightarrow$ turbulent diffusion, due $\left|\tilde{\phi}\right|^2$
- Multi-scale interaction <u>branches</u> thru ES, Magnetic Scattering

The Feedback Loops:



Where Things Stand

- Integro-differential equation for $\bar{\phi}$ evolution in presence specified $|b_{k'}|^2$
- Technically complex...
- $(\nabla_{\parallel} J_{\parallel})^{(3)}$ magnetic torque is clear and novel effect, damping vorticity
- Can formulate perturbation theory $\gamma_k \to \gamma_k^{(0)} + \delta\gamma_k$, in terms quadratic form
- Detailed analysis ongoing ...

Conclusions – Lessons Learned, so far…

- Problem of instability in stochastic field is intrinsically multi-scale and dynamic: $\bar{\phi}$; $\tilde{\phi}$ and $\tilde{b}$
- To maintain $\nabla \cdot \vec{J} = 0$ for prescribed $\tilde{b}_{k'}$ + instability ➔ $\tilde{\phi}$ generated
- Physics: $\nabla_\perp \cdot J_\perp \neq 0$ to maintain $\nabla \cdot \vec{J} = 0$ ➔ Enter electrostatic micro-cells !
- Magnetic vorticity damping is generic to stochastic $\tilde{b}$ + turbulence
- Inertia → Inertia + $\frac{S}{\tau_A}\partial_x |b_r|^2 \partial_x \bar{\phi}$
- FOM : w_I' vs $\left[\left(k_\theta^2/{k_\theta'}^2\right)(\Delta x)^4\right]^{1/4}$ for: $\nabla_\| J_\|^{(2)} \sim \nabla_\| J_\|^{(1)}$

Conclusions – Lessons Learned, so far...

- More generally, for turbulence $\tilde{\phi}$ in stochastic $\tilde{b}$; cannot treat as statistically independent i.e. $\boxed{\langle \tilde{b}\,\tilde{\phi}\,\rangle \neq 0}$
- small scale $\tilde{b}$ leaves 'footprint' on modes

A Look Ahead:

- Complete the analysis – bistability ?
- Collisionless → Alfvenic radiation into network of $\langle \tilde{b}^2 \rangle$

  (c.f. C-C Chen, this meeting)
- Statistical analysis Pdf($\tilde{b}$) → Distribution of Eigenvalues

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